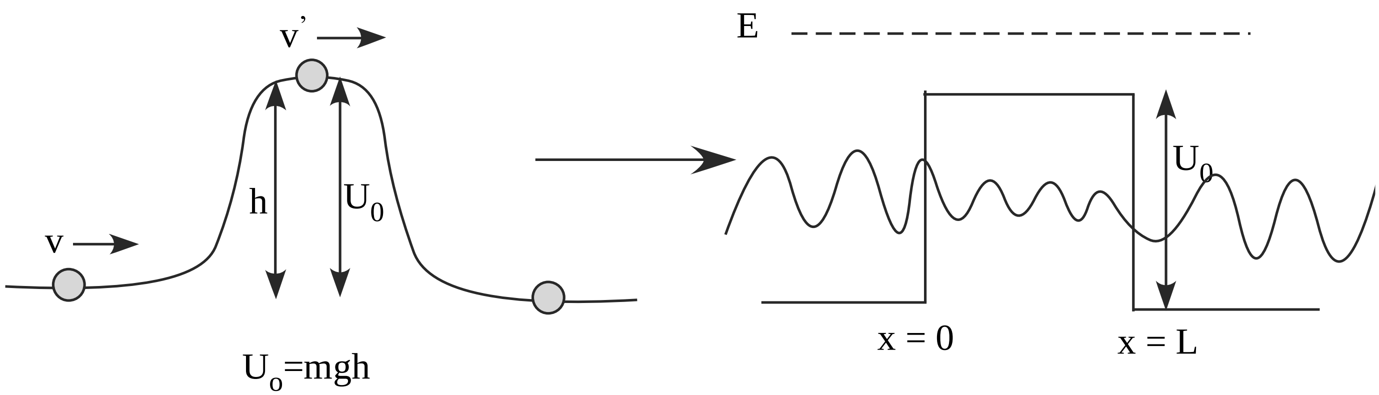
TUNNELING

A. THE SQUARE BARRIER

CASE 1: E > U_0

Consider a bead (particle) of mass m moving with speed *v* on a frictionless wire as shown below. The particle encounters a hill of height *h* as it moves to the right.



Square Barrier (Potential)

1. Classically, if the energy E of the particle is greater than U_0, then the particle will have sufficient energy (E > U_0) to move up the hill and to the top.

$$E = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2E}{m}}$$

2. While moving at the top of the hill the particle has lost kinetic energy and its speed is now given by:

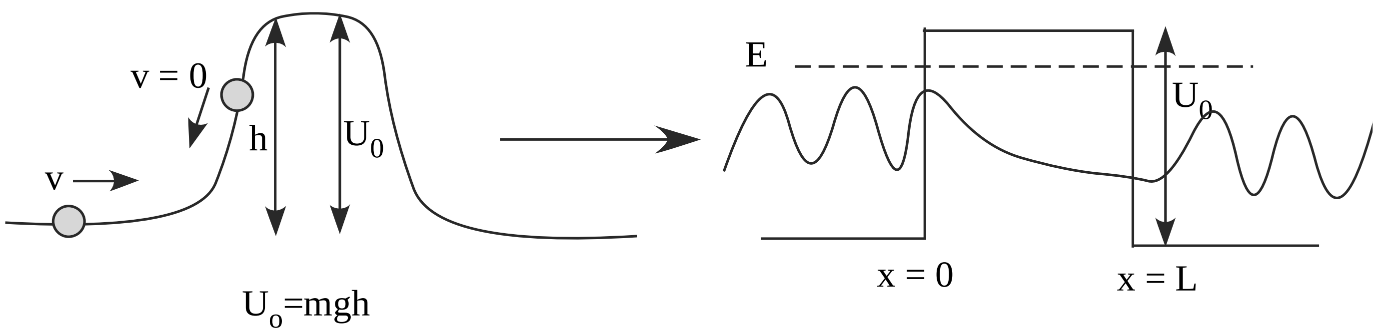
$$E = \frac{1}{2}mv'^2 + U_0 \Rightarrow v' = \sqrt{\frac{2}{m}\left(E - U_0\right)}$$

3. When the particle reaches the other side of the hill it regains its initial energy E and moves with the same initial speed.

4. Classically for E > U_0 all the particles make it across the barrier. The particles are able to penetrate the barrier and thus are transmitted across the barrier.

CASE 1: $E < U_0$

Consider the same particle as before but now with different initial KE.



Square Barrier (Potential)

1. Classically, since the energy E of the particle is less than U_0, then the particle will have insufficient energy ($E < U_0$) to move up the hill and to the top.

$$E\left(\frac{1}{2}mv^2\right) < U_0\,(mgh)$$

2. The particle will move partway up and then back down to the left with its original energy E.

5. Classically for $E < U_0$ all the particles do not make it across the barrier. The particles are reflected at the barrier not having sufficient energy to penetrate across the barrier and thus are all reflected back.

These two cases, for $E > U_0$ and $E < U_0$, divide the space into classically allowed and forbidden regions determined by the energy of the particles.

HOWEVER, according to Quantum Mechanics, there is NO region that is not accessible to the particles, regardless of their initial energy!