## SECTION 2.1 PROBLEM SET: INTRODUCTION TO MATRICES

A vendor sells hot dogs and corn dogs at three different locations. His total sales(in hundreds) for January and February from the three locations are given in the table below.

JANUARY FEBRUARY

HOT DOGS CORN DOGS HOT DOGS CORN DOGS

PLACE I 10 8 8 7

PLACE II 8 6 6 7

PLACE III 6 4 6 5

Represent these tables as 32 matrices J and F, and answer problems 1 - 5.

|  |  |
| --- | --- |
| 1) Determine total sales for the two months, that is, find J + F. | 2) Find the difference in sales, J – F. |
| 3) If hot dogs sell for $3 and corn dogs for $2, find the revenue from the sale of hot dogs and corn dogs. *Hint: Let P be a 21 matrix.  Find (J + F)P.* | 4) If March sales will be up from February by 10%, 15%, and 20% at Place I, Place II, and Place III, respectively, find the expected number of hot dogs and corn dogs to be sold  in March. *Hint: Let R be a 13 matrix with entries 1.10, 1.15, and 1.20. Find M = RF.* |
| 5) Hots dogs sell for $3 and corn dogs sell for $2. Using matrix M that predicts the number of hot dogs and corn dogs expected to be sold in March from problem (4), find the 1x1 matrix that predicts total revenue in March. *Hint: Use 2x1 price matrix P from problem (3) and find MP.* | |

***SECTION 2.1 PROBLEM SET: INTRODUCTION TO MATRICES***

Determine the sums and products in problems 6-13. Given the matrices A, B, C, and D as follows:

A =  B =  C =  D = 

|  |  |
| --- | --- |
| 6) 3A – 2B | 7) AB |
| 8) BA | 9) AB + BA |
| 10) A2 | 11) 2BC |
| 12) 2CD + 3AB | 13) A2B |

***SECTION 2.1 PROBLEM SET: INTRODUCTION TO MATRICES***

|  |  |
| --- | --- |
| 14) Let E = and F = ,  find EF. | 15) Let E = and F = ,  find FE. |
| 16) Let G = H = , find GH. | 17) Let G = H =  Explain why the product HG does not exist. |

Express the following systems as AX = B, where A, X, and B are matrices.

|  |  |
| --- | --- |
| 18) 4x – 5y = 6  5x – 6y = 7 | 19) x - 2y + 2z = 3  x - 3y + 4z = 7  x - 2y - 3z = -12 |
| 20) 2x + 3z = 17  3x – 2y = 10  5y + 2z = 11 | 21) x + 2y + 3z + 2w = 14  x – 2y – z = – 5  y – 2z + 4w = 9  x + 3z + 3w = 15 |

## SECTION 2.2 PROBLEM SET: SYSTEMS OF LINEAR EQUATIONS

Solve the following by the Gauss-Jordan Method. Show all work.

|  |  |
| --- | --- |
| 1) x + 3y = 1  2x – 5y = 13 | 2) x – y – z = –1  x – 3y + 2z = 7  2x – y + z = 3 |
| 3) x + 2y + 3z = 9  3x + 4y + z = 5  2x – y + 2z = 11 | 4) x + 2y = 0  y + z = 3  x + 3z = 14 |

***SECTION 2.2 PROBLEM SET: SYSTEMS OF LINEAR EQUATIONS***

Solve the following by the Gauss-Jordan Method. Show all work.

|  |  |
| --- | --- |
| 5) Two apples and four bananas cost $2.00 and three apples and five bananas cost $2.70. Find the price of each. | 6) A bowl of corn flakes, a cup of milk, and an egg provide 16 grams of protein. A cup of milk and two eggs provide 21 grams of protein. Two bowls of corn flakes with two cups of milk provide 16 grams of protein. How much protein is provided by one unit of food? |
| 7) x + 2y = 10  y + z = 5  z + w = 3  x + w = 5 | 8) x + w = 6  2x + y + w = 16  x – 2z = 0  z + w = 5 |

## SECTION 2.3 PROBLEM SET: SYSTEMS OF LINEAR EQUATIONS – SPECIAL CASES

Solve the following inconsistent or dependent systems by using the Gauss-Jordan method.

|  |  |
| --- | --- |
| 1) 2x + 6y = 8  x + 3y = 4 | 2) The sum of digits of a two digit number is 9. The sum of the number and the number obtained by interchanging the digits is 99.  Find the number. |
| 3) 2x – y = 10  – 4x + 2y = 15 | 4) x + y + z = 6  3x + 2y + z = 14  4x + 3y + 2z = 20 |
| 5) x + 2y – 4z = 1  2x – 3y + 8z = 9 | 6) Jessica has a collection of 15 coins consisting of nickels, dimes and quarters. If the total worth of the coins is $1.80, how many are  there of each? Find all three solutions. |

***SECTION 2.3 PROBLEM SET: SYSTEMS OF LINEAR EQUATIONS – SPECIAL CASES***

Solve the following inconsistent or dependent systems by using the Gauss-Jordan method.

|  |  |
| --- | --- |
| 7) A company is analyzing sales reports for three products: products X, Y, Z. One report shows that a combined total of 20,000 of items X, Y, and Z were sold. Another report shows that the sum of the number of item Z sold and twice the number of item X sold equals 10,000. Also item X has 5,000 more items sold than item Y.  Are these reports consistent? | 8) x + y + 2z = 0  x + 2y + z = 0  2x + 3y + 3z = 0 |
| 9) Find three solutions to the following system  of equations.  x + 2y + z = 12  y = 3 | 10) x + 2y = 5  2x + 4y = k  For what values of k does this system of equations have  a) No solution?  b) Infinitely many solutions? |
| 11) x + 3y – z = 5 | 12) Why is it not possible for a linear system to have exactly two solutions? Explain geometrically. |

## SECTION 2.4 PROBLEM SET: INVERSE MATRICES

In problems 1- 2, verify that the given matrices are inverses of each other.

|  |  |
| --- | --- |
| 1) | 2) |

In problems 3- 6, find the inverse of each matrix by the row-reduction method.

|  |  |
| --- | --- |
| 3) | 4) |

***SECTION 2.4 PROBLEM SET: INVERSE MATRICES***

In problems 5- 6, find the inverse of each matrix by the row-reduction method.

|  |  |
| --- | --- |
| 5) | 6) |

Problems 7-10: Express the system as AX = B; then solve using matrix inverses found in problems 3-6.

|  |  |
| --- | --- |
| 7) 3x – 5y = 2  –x + 2y = 0 | 8) x + 2z = 8  y + 4z = 8  z = 3 |

***SECTION 2.4 PROBLEM SET: INVERSE MATRICES***

Problems 9-10: Express the system as AX = B; then solve using matrix inverses found in problems 3-6.

|  |  |
| --- | --- |
| 9) x + y – z = 2  x + z = 7  2x + y + z = 13 | 10) x + y + z = 2  3x + y = 7  x + y + 2z = 3 |
| 11) Why is it necessary that a matrix be a square matrix for its inverse to exist? Explain by relating the matrix to a system of equations. | 12) Suppose we are solving a system AX = B by the matrix inverse method, but discover A has no inverse. How else can we solve this system? What can be said about the solutions of this system? |

## SECTION 2.5 PROBLEM SET: APPLICATION OF MATRICES IN CRYPTOGRAPHY

In problems 1- 8, the letters A to Z correspond to the numbers 1 to 26, as shown below, and a space is represented by the number 27.

A B C D E F G H I J K L M

1 2 3 4 5 6 7 8 9 10 11 12 13

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

N O P Q R S T U V W X Y Z

14 15 16 17 18 19 20 21 22 23 24 25 26

In problems 1 - 2, use the matrix A, given below, to encode the given messages.

A =

In problems 3 - 4, decode the messages that were encoded using matrix A.  
Make sure to consider the spaces between words, but ignore all punctuation. Add a final space if necessary.

|  |  |
| --- | --- |
| 1) Encode the message: WATCH OUT! | 2) Encode the message: HELP IS ON THE WAY. |
| 3) Decode the following message:  64 23 102 41 82 32 97 35 71 28 69 32 | 4) Decode the following message:  105 40 117 48 39 19 69 32 72 27 37  15 114 47 |

***SECTION 2.5 PROBLEM SET: APPLICATION OF MATRICES IN CRYPTOGRAPHY***

In problems 5 - 6, use the matrix B, given below, to encode the given messages.

B = 

In problems 7 - 8, decode the messages that were encoded using matrix B.

Make sure to consider the spaces between words, but ignore all punctuation. Add a final space if necessary.

|  |  |
| --- | --- |
| 5) Encode the message using matrix B:  LUCK IS ON YOUR SIDE. | 6) Encode the message using matrix B:  MAY THE FORCE BE WITH YOU. |
| 7) Decode the following message that was encoded using matrix B:  8 23 7 4 47 – 2 15 102 –12 20 58 15 27 80 18 12 74 –7 | 8) Decode the following message that was encoded using matrix B:  12 69 – 3 11 53 9 5 46 –10 18 95 – 9 25 107 4 27 76 22 1 72 –26 |

## SECTION 2.6 PROBLEM SET: APPLICATIONS – LEONTIEF MODELS

1) Solve the following homogeneous system.

x + y + z = 0

3x + 2y + z = 0

4x + 3y + 2z = 0

2) Solve the following homogeneous system.

x – y – z = 0

x – 3y + 2z = 0

2x – 4y + z = 0

3) Chris and Ed decide to help each other by doing repairs on each others houses. Chris is a carpenter, and Ed is an electrician. Chris does carpentry work on his house as well as on Ed's house. Similarly, Ed does electrical repairs on his house and on Chris' house. When they are all finished they realize that Chris spent 60% of his time on his own house, and 40% of his time on Ed's house. On the other hand Ed spent half of his time on his house and half on Chris's house. If they originally agreed that each should get about a $1000 for their work, how much money should each get for their work?

4) Chris, Ed, and Paul decide to help each other by doing repairs on each others houses. Chris is a carpenter, Ed is an electrician, and Paul is a plumber. Each does work on his own house as well as on the others houses. When they are all finished they realize that Chris spent 30% of his time on his own house, 40% of his time on Ed's house, and 30% on Paul's house. Ed spent half of his time on his own house, 30% on Chris' house, and remaining on Paul's house. Paul spent 40% of the time on his own house, 40% on Chris' house, and 20% on Ed's house. If they originally agreed that each should get about a $1000 for their work, how much money should each get for their work?

***SECTION 2.6 PROBLEM SET: APPLICATIONS – LEONTIEF MODELS***

5) Given the internal consumption matrix A, and the external demand matrix D as follows.

A = D =

Solve the system using the open model: X = AX + D or X = (I – A)–1D

6) Given the internal consumption matrix A, and the external demand matrix D as follows.

A = D =

Solve the system using the open model: X = AX + D or X = (I – A)–1D

7) An economy has two industries, farming and building. For every $1 of food produced, the farmer uses $.20 and the builder uses $.15. For every $1 worth of building, the builder uses $.25 and the farmer uses $.20. If the external demand for food is $100,000, and for building $200,000, what should be the total production for each industry in dollars?

***SECTION 2.6 PROBLEM SET: APPLICATIONS – LEONTIEF MODELS***

8) An economy has three industries, farming, building, and clothing. For every $1 of food produced, the farmer uses $.20, the builder uses $.15, and the tailor $.05. For every $1 worth of building, the builder uses $.25, the farmer uses $.20, and the tailor $.10. For every $1 worth of clothing, the tailor uses $.10, the builder uses $.20, the farmer uses $.15. If the external demand for food is $100 million, for building $200 million, and for clothing $300 million, what should be the total production for each in dollars?

9) Suppose an economy consists of three industries F, C, and T. The following table gives information about the internal use of each industry's production and external demand in dollars.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | F | C | T | Demand | Total |
| F | 30 | 10 | 20 | 40 | 100 |
| C | 20 | 30 | 20 | 50 | 120 |
| T | 10 | 10 | 30 | 60 | 110 |

Find the proportion of the amounts consumed by each of the industries; that is, find the matrix A.

10) If in problem 9, the consumer demand for F, C, and T becomes 60, 80, and 100, respectively, find the total output and the internal use by each industry to meet that demand.

## SECTION 2.7 PROBLEM SET: CHAPTER REVIEW

1) To reinforce her diet, Mrs. Tam bought a bottle containing 30 tablets of Supplement A and a bottle containing 50 tablets of Supplement B. Each tablet of supplement A contains 1000 mg of calcium, 400 mg of magnesium, and 15 mg of zinc, and each tablet of supplement B contains 800 mg of calcium, 500 mg of magnesium, and 20 mg of zinc.

a) Represent the amount of calcium, magnesium and zinc in each tablet as a 23 matrix.

b) Represent the number of tablets in each bottle as a row matrix.

c) Use matrix multiplication to find the total amount of calcium, magnesium, and zinc in both bottles.

2) Let matrix A =and B= . Find the following.

a) b) 3A – 2B

3) Let matrix C = and D = . Find the following.

a) 2(C – D) b) C – 3D

4) Let matrix E = and F = . Find the following.

a) 2EF b) 3FE

5) Let matrix G = and H =. Find the following.

a) 2GH b) HG

6) Solve the following systems using the Gauss-Jordan Method.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a) | x | + | 3y | – | 2z | = | 7 |  | b) | 2x | – | 4y | + | 4z | = | 2 |
|  | 2x | + | 7y | – | 5z | = | 16 |  |  | 2x | + | y | + | 9z | = | 17 |
|  | x | + | 5y | – | 3z | = | 10 |  |  | 3x | – | 2y | + | 2z | = | 7 |

7) An apple, a banana and three oranges or two apples, two bananas, and an orange, or four bananas and two oranges cost $2. Find the price of each.

8) Solve the following systems. If a system has an infinite number of solutions, first express the solution in parametric form, and then determine one particular solution.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a) | x | + | y | + | z | = | 6 |  | b) | x | + | y | + | 3z | = | 4 |
|  | 2x | – | 3y | + | 2z | = | 12 |  |  | x |  |  | + | z | = | 1 |
|  | 3x | – | 2y | + | 3z | = | 18 |  |  | 2x | – | y |  |  | = | 2 |

9) Elise has a collection of 12 coins consisting of nickels, dimes and quarters. If the total worth of the coins is $1.80, how many are there of each? Find all possible solutions.

***SECTION 2.7 PROBLEM SET: CHAPTER REVIEW***

10) Solve the following systems. If a system has an infinite number of solutions, first express the solution in parametric form, and then find a particular solution.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a) | x | + | 2y |  |  | = | 4 |  | b) | | x | | – | 2y | | | + | 2z | | | = | 1 | |
|  | 2x | + | 4y |  |  | = | 8 |  |  | | 2x | | – | 3y | | | + | 5z | | | = | 4 | |
|  | 3x | + | 6y | – | 3z | = | 3 |  | |  | |  | | |  |  | | |  |  | | |  | |  |

11) Solve the following systems. If a system has an infinite number of solutions, first express the solution in parametric form, and then provide one particular solution.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a) | 2x | + | y | – | 2z | = | 0 |  | b) | 3x | + | 4y | – | 3z | = | 5 |
|  | 2x | + | 2y | – | 3z | = | 0 |  |  | 2x | + | 3y | – | z | = | 4 |
|  | 6x | + | 4y | – | 7z | = | 0 |  |  | x | + | 2y | + | z | = | 1 |

12) Find the inverse of the following matrices: a) b)

13) Solve the following systems using the matrix inverse method.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a) | 2x | + | 3y | + | z | = | 12 |  | b) | x | + | 2y | – | 3z | + | w | = | 7 |
|  | x | + | 2y | + | z | = | 9 |  |  | x |  |  | – | z |  |  | = | 4 |
|  | x | + | y | + | z | = | 5 |  |  | x | – | 2y | + | z |  |  | = | 0 |
|  |  |  |  |  |  |  |  |  |  |  |  | y | – | 2z | + | w | = | –1 |

14) Use matrix A to encode the following messages.   
The space between the letters is represented by the number 27,   
and all punctuation is ignored.

A =

a) TAKE IT AND RUN. b) GET OUT QUICK.

15) Decode the following messages that were encoded using matrix A in the above problem.

a) 44, 71, 15, 18, 27, 1, 68, 82, 27, 69, 76, 27, 19, 33, 9

b) 37, 64, 15, 36, 54, 15, 67, 75, 20, 59, 66, 27, 39, 43, 12

16) Chris, Bob, and Matt decide to help each other study during the final exams. Chris's favorite subject is chemistry, Bob loves biology, and Matt knows his math. Each studies his own subject as well as helps the others learn their subjects. After the finals, they realize that Chris spent 40% of his time studying his own subject chemistry, 30% of his time helping Bob learn chemistry, and 30% of the time helping Matt learn chemistry. Bob spent 30% of his time studying his own subject biology, 30% of his time helping Chris learn biology, and 40% of the time helping Matt learn biology. Matt spent 20% of his time studying his own subject math, 40% of his time helping Chris learn math, and 40% of the time helping Bob learn math. If they originally agreed that each should work about 33 hours, how long did each work?

17) As in the previous problem, Chris, Bob, and Matt decide to not only help each other study during the final exams, but also tutor others to make a little money. Chris spends 30% of his time studying chemistry, 15% of his time helping Bob with chemistry, and 25% helping Matt with chemistry. Bob spends 25% of his time studying biology, 15% helping Chris with biology, and 30% helping Matt. Similarly, Matt spends 20% of his time on his own math, 20% helping Chris, and 20% helping Bob. If they spend respectively, 12, 12, and 10 hours tutoring others, how many total hours are they going to end up working?