

Name: \_\_\_\_\_

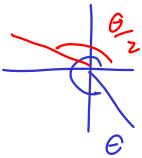
Score: \_\_\_\_\_

- Please show all work in the space provided for each problem and circle your final answer when appropriate. **No credit will be awarded if no work is shown.**
- **No credit may be given for a decimal approximation in a problem which asks for an exact answer.**

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**Problem 1.** [4+4+4+4+4=20 points] Circle either **True** or **False** for each of the following statements.

(a) If  $\theta$  is in Quadrant 4, then  $\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos\theta}{2}}$



True  False

(b) Since  $\tan(\pi) = 0$  and  $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$ , we know  $\frac{2 \tan\left(\frac{\pi}{2}\right)}{1 - \tan^2\left(\frac{\pi}{2}\right)} = 0$

$\tan\left(\frac{\pi}{2}\right) \text{ DNE}$

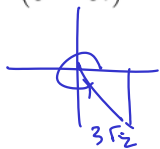
True False

(c)  $3(\cos 190^\circ + i \sin 190^\circ) \cdot 4(\cos 200^\circ + i \sin 200^\circ) = 12(\cos 30^\circ + i \sin 30^\circ)$

$190^\circ + 200^\circ = 390^\circ \text{ co-terminal w/ } 30^\circ$

True  False

(d)  $(3 - 3i)^4 = 81 + 81i$



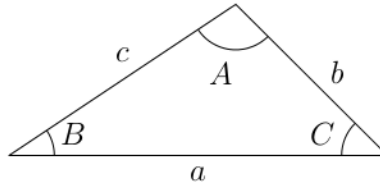
$\left[3\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\right]^4 = 18^2(-1)$

True False

(e)  $z = 115 + \sqrt{3}i$  has 4 fourth roots.

True  False

Problems 2 and 3 refer to the labels in the triangle below. Answers should be rounded to **two decimal places** and angle measures should be given in degrees. Please note that this triangle is not drawn to scale.



**Problem 2.** [8+4=12 points]

(a) Given  $a = 10$ ,  $b = 5$  and  $c = 7$ , find  $A$ ,  $B$  and  $C$  for **all possible triangles**.

$$\cos A = \frac{25 + 49 - 100}{70}$$

$$A = 111.80^\circ$$

$$\frac{\sin B}{5} = \frac{\sin 111.80^\circ}{10}$$

$$B = 27.66^\circ$$

$$C = 40.54^\circ$$

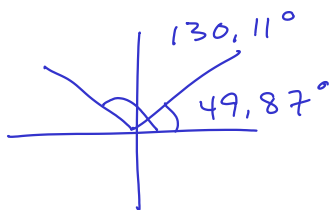
(b) Find the area of the triangle(s) in part (a).

$$\text{Area} = \sqrt{11(6)(4)} = 16.25$$

**Problem 3.** [12 points] Given  $a = 8$ ,  $b = 6$  and  $B = 35^\circ$ , find  $A$ ,  $C$  and  $c$  for **all possible triangles**.

$$\frac{\sin A}{8} = \frac{\sin 35^\circ}{6}$$

$$\sin A = 0.7648$$



$$A = 49.87^\circ$$

$$C = 95.13^\circ$$

$$c = 10.42$$

$$A = 130.11^\circ$$

$$C = 14.89^\circ$$

$$c = 2.69$$

$$\frac{c}{\sin 95.13^\circ} = \frac{6}{\sin 35^\circ}$$

$$\frac{c}{\sin 14.89^\circ} = \frac{6}{\sin 35^\circ}$$

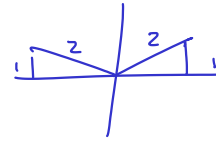
**Problem 4. [12 points]** Find all solutions (in radians) for  $\sin(2\theta) - \cos(\theta) = 0$  on the interval  $[0, 2\pi)$ . Be sure to give the **exact** solution(s).

$$2 \sin \theta \cos \theta - \cos \theta = 0$$

$$\cos \theta (2 \sin \theta - 1) = 0$$

$$\cos \theta = 0$$

$$\sin \theta = \frac{1}{2}$$



$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

**Problem 5. [4 points]** Find the **exact** value of  $\cos(105^\circ)$ .

$$\cos(45^\circ + 60^\circ)$$

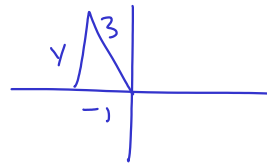
$$= \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} - \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} = \frac{1 - \sqrt{3}}{2\sqrt{2}}$$

**Problem 6. [8+8=16 points]** Given  $\sec \theta = -3$  and  $\theta$  is in Quadrant 2, find the **exact** value for the following.

(a)  $\cos(2\theta)$

$$\cos \theta = -\frac{1}{3}$$



$$1 + y^2 = 9$$

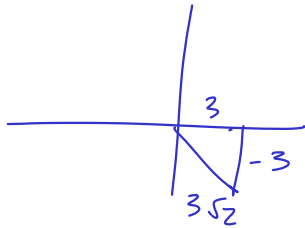
$$y = \sqrt{8}$$

$$\cos^2 \theta - \sin^2 \theta = \frac{1}{9} - \frac{8}{9} = \frac{-7}{9}$$

(b)  $\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1}{2}(1 + \cos \theta)}$

$$\sqrt{\frac{1}{2}\left(1 - \frac{1}{3}\right)} = \frac{1}{\sqrt{3}}$$

**Problem 7. [4 points]** Write  $z = 3 - 3i$  in trigonometric form.



$$z = 3\sqrt{2} \left( \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

**Problem 8. [4+4+4+4+4=20 points]** Answer the following for the vectors  $\mathbf{v} = \langle 2, -4 \rangle$  and  $\mathbf{u} = \langle 1, 5 \rangle$ .

(a)  $\|\mathbf{v}\|$   $\sqrt{20}$

(b)  $2\mathbf{v} - 3\mathbf{u}$   $\langle 4, -8 \rangle + \langle -3, -15 \rangle = \langle 1, -23 \rangle$

(c)  $\mathbf{u} \cdot \mathbf{v}$   $2 - 20 = -18$

(d) The angle between  $\mathbf{v}$  and  $\mathbf{u}$   $\cos \theta = \frac{-18}{\sqrt{20 \cdot 26}}$

$$\theta = 142.13^\circ$$

(e)  $\text{proj}_{\mathbf{v}} \mathbf{u}$

$$\frac{-18}{20} \langle 2, -4 \rangle = \frac{-9}{10} \langle 2, -4 \rangle = \left\langle \frac{-9}{5}, \frac{18}{5} \right\rangle$$

